The Wing Twist Maximization of Aeroelastic Car-Spoiler

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Abstract

The rear spoiler of car is employed to control lift of car with producing the negative lift or downforce. This effect makes the driving safer. The conventional spoiler is designed to be stiff and strong to reduce structural deformation and aeroelastic problem. The angle of attack of such a spoiler is set to a high angle to get more downforce at the high speed. However, the downforce at the low speed is still created and it is not needed. This paper presents a new idea to take advantage of wing twist at the high speed whilst there is smaller angle of attack at the low speed. The technology of Multidisciplinary Design Optimization applied to the airfoil spoiler with one fixed spar to allow the structural deformation whereas the structural and aeroelastic failures were avoided. The optimization problem was set up to maximize the wing twist angle of the spoiler with varying the thickness of spoiler skin and spar, the height of a trapezoid spar subject to structural and aeroelastic constraints such as deflection, stress, divergence and flutter. Population Based Incremental Learning (PBIL) was selected to solve such a problem. The structural and aeroelastic analysis are performed in MATLAB program with developing Finite Element and PBIL algorithm. The results are compared with a general spoiler using a flat plate. The outcomes demonstrated that the airfoil spoiler provided the higher angle of attack at high speed and the lower angle of attack at low speed whilst its deflection was small when their results were compared with the results of the flat plate spoiler. Therefore, this paper shows the success of employing MDO to search for the maximum twist angle of aeroelastic spoiler so that the spoiler wing deformation is taken in a positive way.

Keywords*:* Car Spoiler, Aeroelastic Wing, Optimization

Nomenclature

θ = Angle of attack at wing tip
ν = Nodal displacement
f = Objective function
g = Inequality constraint
x = Design variable
$\mathbf{x}$ = Vector of design variables

3AS = Active Aeroelastic Structures
AIS = Active Internal Structures or Adaptive Internal Structures
AOA = Angle of Attack
DOF = Degree of Freedom
FE = Finite Element
FEA = Finite Element Analysis

MDO = Multidisciplinary Design Optimization
R_i = Ratio of the nodal stress to the allowable stress at the i^{th} node
V_{cr} = Critical speed.

1. Introduction

There are many efforts to design the speed of car approaching the speed of the race car. The effects of lift and pitching moment [1] are larger on the directional stability of car when its speed is over 100 km/hr. To solve such a problem, car spoilers at the front and back are employed to control lift and pitching moment at the high speed of car so that the negative lift or downforce is produced. This effect makes the driving safer. In general, the rear spoiler has two types: the ducktail spoiler and the wing spoiler.

This paper concentrates on the wing spoiler. In general, the structure of spoiler is very stiff to reduce the structural deformation and aeroelastic failures of the structure. Additionally, it is also designed with the high angle of attack (AOA) to obtain more downforce. However, this concept is not desired at the low speed of car because the downforce is still happened. This effect increases a car load. That means that the car takes more drag and more fuel consumption in the driving due to this aerodynamic load.

A number of research programmes on Active Aeroelastic Structures (3AS) concepts [2-4] for modern aircrafts have been studied to allow structural deflection to enhance aerodynamic performance. This means that aeroelasticity is considered in a positive manner by allowing structural deformation. However, the attempts to reduce the structural stiffness can lead to aeroelastic problems. In 2003, Cooper and Amprikidis [5, 6] first proposed Adaptive Internal Structures (AIS) concepts to control the structural deflection and aeroelastic behaviours. Their work showed that it was possible to change the bending and torsional stiffness in order to control the aeroelastic behavior. The basic idea used in AIS was to make the use of aerodynamic forces acting upon the wing to provide the moment due to the lift acting on the aerodynamic center (a.c.) and the distance between the a.c. and e.a. (elastic axis or torsion axis) by changing the position of e.a. with using the smart spar concepts of moving and rotating spars in AIS. In recent year, their works confronted the difficulty to twist the wing structure using the smart spar concepts because the structure was too stiff. Attempts to improve the performance and reduce the structural weight can no longer be found by conventional design. Kittipichai and Cooper [7, 8] exploited the technology of Multidisciplinary Design Optimization (MDO) to get better performance and obtain the minimum mass of AIS whilst the structural and aeroelastic problems were avoided. Like the idea of AIS which allowed the structural deformation, Kittipichai [9] has recently proposed the idea to reduce the mass of the aeroelastic spoiler wing and allow the structural twist whilst the structural and aeroelastic problem were avoided.

To use the idea of AIS and take advantage of spoiler wing twist, this paper will propose the idea to maximize the wing twist. The structure of the spoiler will consist of ten ribs and one fixed spar. The model of wing spoiler as shown in Fig. 1 will be studied. However, it may be slightly different from the wing spoiler which used in the commercial spoiler. That is the distance between both legs of the model narrow than that of the commercial spoiler to allow the spoiler wing to deform easily. Additionally, the angle of attack of the model is smaller than the commercial spoiler to reduce the problem from unnecessary downforce at the low speed.

This paper starts with analyzing a flat plate spoiler as shown in Fig. 1 to study the change of wing twist, deflection, natural frequency, divergence and flutter of the structure. The structure is made of aluminium. Finally, the airfoil spoiler as shown in Fig. 2 is studied and its result is compared with the flat spoiler.

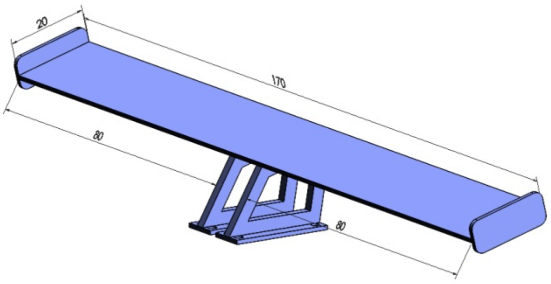


Fig. 2 A full scale of a flat plate spoiler

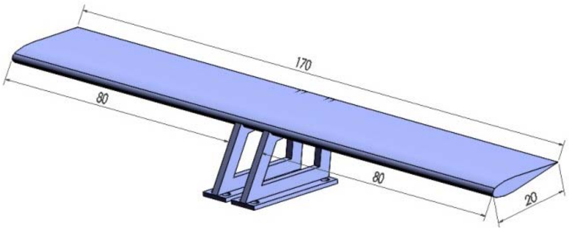


Fig. 1 A full scale of wing spoiler model

2. Flexible spoiler using flat plate

This part is a primary study to analyze an aeroelastic spoiler in the structural deformation. As shown in Fig. 2, the structural model of spoiler is a flat plate which is made of Aluminum. The size of spoiler is 0.2(W) × 1.7(L) × 0.003(T) m and the distance between both legs is 0.1 m. The section of plate between both legs is considered as the rigid body. Due to symmetry, only a part of spoiler wing can be considered to investigate structural deformation. Then, the size of a flexible wing becomes 0.2 (W) × 0.8(L) × 0.003 (T) m. Using Finite Element Analysis (FEA), the flexible wing was modeled by using the four node quadrilateral shell element with six Degrees of Freedom (DOFs) per node. Fig. 8 showed FE model of the flexible wing consisting of 45 nodes and 32 elements. It should be noted that the nodal points at the root of wing are fixed whereas others are left free.

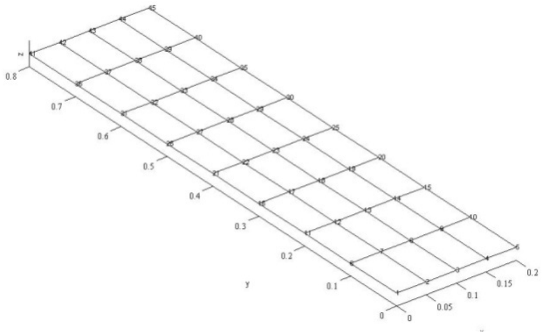


Fig. 3 Finite Element model of flexible wing

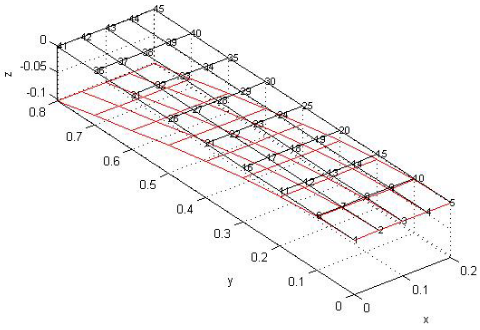
As shown in Fig. 3, the body of spoiler wing is similar to an aircraft wing therefore the basic principles from aircraft aerodynamics can be applied to the structure. Aerodynamic forces were obtained by using strip theory. Four strips of aerodynamic forces along the wing were applied for both steady and unsteady aerodynamics. Both static and dynamic aeroelasticity such as displacement, divergence and flutter, etc. were determined by using FE method. The structural and aeroelastic analysis were done in MATLAB program with developing FE code to determine natural frequency, divergence and flutter including tip twist, deflection and downforce of the structure at various speeds from 0 to 200 km/hr and various AOA at 8 degree.

From the results of FEA for the aluminium wing, the divergence and flutter speeds were 129.8 m/s (or 467.2 km/hr) and 108.5 m/s (or 390.7 km/hr), respectively. Fig. 4 shows 4 natural frequencies and mode shapes: the 1st bending at 3.9 Hz, the 2nd bending at 25.3 Hz, the 1st torsion at 31.1 Hz, and the 3rd bending at 76.4 Hz. Fig. 5 shows the results of (a) changes of tip twist angle, (b) changes of maximum structural deflection, (c) change of downforce on a rigid wing, a flexible wing and difference of downforce between the flexible and rigid wing against car speed. When the car speed increases, these values will be increased as well. Particularly, when the speed is more than 60 km/hr, they will be dramatically increased. As shown in Fig. 5 (c), it is obviously noticed that the downforce on the flexible wing is more than that on the rigid wing when the speed is more than 100 km/hr. The increase of downforce on the flexible wing has the cause from the change of AOA on the local wing due to aerodynamic force. This result shows that the downforce can be significantly increased when the car speeds up more than 100 km/hr. Nevertheless, the deflection is very large. For example, at the speed of 160 km/hr and AOA at wing tip of 9.7 degree, the maximum deflection is about 15.6 cm. This shows that the structure has a large deflection and it is not needed due to the limit of space between the spoiler wing and the back of car.

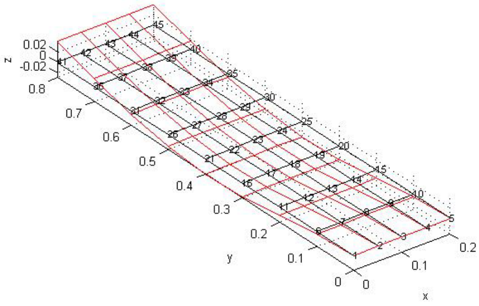
To solve the problem of large deflection and take advantage of wing twist at the high speed, an airfoil spoiler is introduced. However, it cannot be designed with the conventional method to receive the optimal design with taking advantage of the wing twist and avoiding the large deflection and structural failures. A Multidisciplinary Design Optimization (MDO) strategy is needed to obtain the maximum tip twist angle of the flexible spoiler-wing.

3. Optimization Strategies

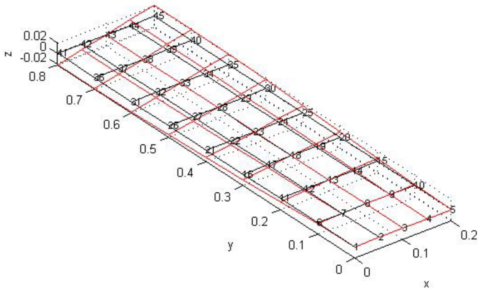
Nowadays, several methods have been proposed to solve the optimization problem in which the wing twist is maximized subject to displacement and stress constrains. Here, Population Based Incremental Learning (PBIL) as an Evolutionary Algorithm (EA) is used to search for the maximum tip twist angle subject to structural and aeroelastic constraints to prevent the structural failure and aeroelastic phenomena. PBIL [10, 11] is a statistical approach of combining genetic algorithms and competitive learning, and defines the genetic population base through the use of a probability vector. The idea of PBIL is similar to Genetic Algorihtm (GAs) [12, 13] but PBIL was the most consistent and the fastest algorithm [14]. PBIL removes the genetic operations from the GAs and creates offspring in each population through a probability vector. With the population being in the binary strings, the probability of each bit in the binary strings is used to create the new solutions. The procedure is stopped when convergence is reached.



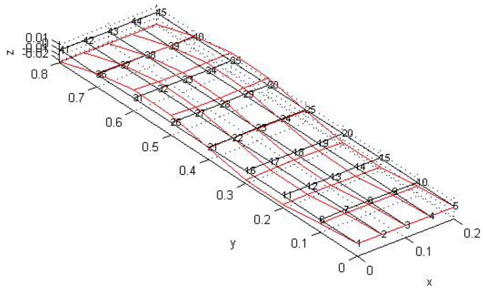
(a) 1st bending at 3.9 Hz



(b) 2nd bending at 25.3 Hz



(c) 1st torsion at 31.1 Hz



(d) 3rd bending at 76.4 Hz

Fig. 4 Four mode shapes of flat plate wing

For the constrained maximization problem, if the objective f(x) and constraint functions $g_i(x)$ are nonlinear function, the general mathematical model for the minimization problem is defined as follows:

$$\text{Min} \quad f(\mathbf{x}) = f(x_1, x_2, x_3, \ldots, x_n) \qquad (1)$$

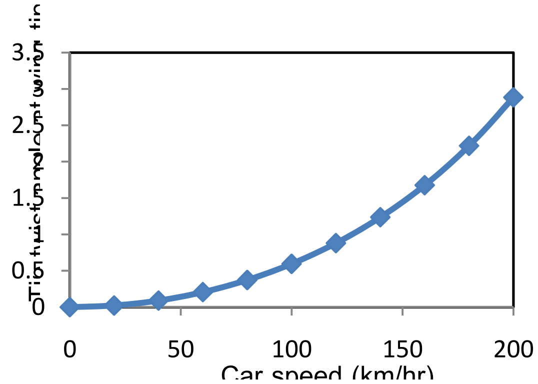
subject to inequality constraints

$$g_i(\mathbf{x}) = g_i(x_1, x_2, x_3, \ldots, x_n) \leq 0; \quad i=1 \text{ to } m \qquad (2)$$

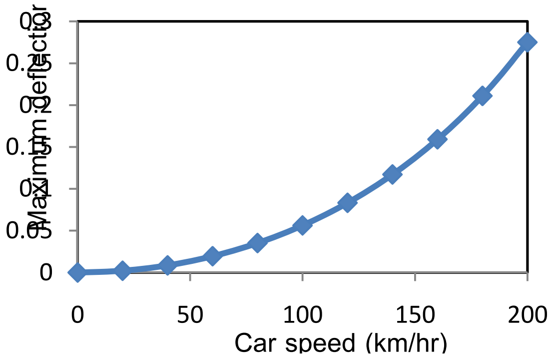
and bounds on design variables

$$x_{l,i} \leq x_i \leq x_{u,i}; \quad i = 1 \text{ to } n \qquad (3)$$

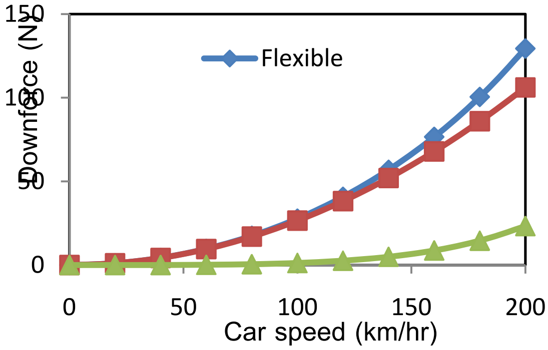
where $f(\mathbf{x})$ is an objective function and $\mathbf{x}$ is a vector of design variables. The variable $x_{l,i}$ and $x_{u,i}$ are the lower and upper bound constraint of the i^{th} variable.



(a) Changes of tip twist angle



(b) Changes of maximum deflection



(c) Changes of downforce on a flexible and rigid semi-wing and difference of both values

Fig. 5 Results of FEA at various car speeds for a flat plate spoiler.

To solve the maximization problem, the objective function can be treated as a minimized function by multiplying by a negative sign to the objective function and Eqs. (2)-(3) are still used.

3.1 Algorithm of Population Based Incremental Learning

As above discussion, PBIL method allows the genetic population based upon a probability vector. The population is in the form of the binary strings {0, 1} or chromosomes. In each generation, the probability value in each bit of string is normally considered to represent the probability of generating a binary bit of either 0 or 1. PBIL algorithm starts with randomly generating a population with all probability values of 0.5. The fitness of each string is evaluated from the objective function to determine the best string. The probability value in each vector position is updated as follows

$$P_i' = P_i(1.0 - LR) + B_i(LR) \tag{4}$$

where P_i' is the updated probability of the i^{th} bit position of the population, P_i is the current probability of the i^{th} bit position, B_i is the i^{th} bit value of the best string and LR is the learning rate. The value of learning rate is between 0 and 1 [15] and can be fixed or varied learning rate. In addition, mutation can be applied to the probability vector before the fitness of each string is evaluated. The mutation probability vector is given as follows

$$P_i' = P_i'(1 - \text{mut\_shift}) + \text{rand}(0.0 \text{ or } 1.0).(\text{mut\_shift}) \tag{5}$$

where mut_shift is the amount for mutation to affect the probability vector. The best string in the current population is kept for the next generation. The process is continued until the criteria are reached. The PBIL algorithm is illustrated in Fig. 6.

Algorithm

1. Initialize the probability vector with all probability values of 0.5.
2. Randomly generate an initial population of N strings using the probability vector.
3. Evaluate the fitness of each string from the objective function.
4. Sort the order of strings based upon their fitness values to determine the best string.
5. Update the probability vector in each vector position by using Eq. (4).
6. Apply the mutation to the probability vector with the predefined probability by using Eq. (5).
7. Stop procedure if the criteria are satisfied; otherwise, go to step 2.

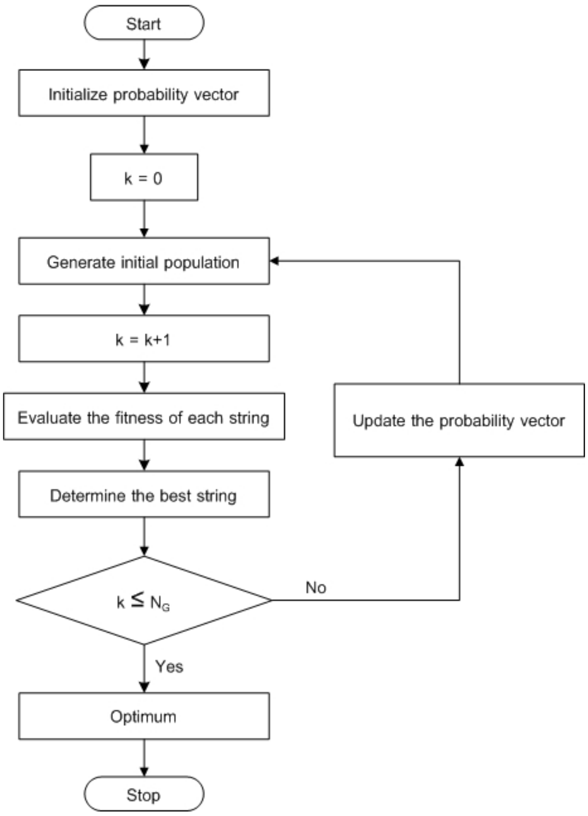


Fig. 7 Algorithm of PBIL

3.2 Penalty Function

Population Based Incremental Learning is used to solve unconstrained optimization problems. Unfortunately, this work is the constrained optimization problem. To deal such a problem, the constrained optimization problem has to be transformed to an unconstrained optimization problem by adding a penalty term of the constraints to the objective function. For a minimization problem as in Eq. (1), the penalized objective function is typically expressed as

$$\text{Min} \quad f_p(\mathbf{x}) = f(\mathbf{x}) + p(\mathbf{x}) \qquad (6)$$

where f is the unpenalized objective function and p is the exterior penalty function. To deal the constrained optimization, many penalty concepts [13, 16] such as dead penalty, static penalty, and dynamic penalty are proposed. Here, the static penalty is selected to solve such a problem because it is a simple way. The penalized objective function in Eq. (6) with m constraints can be rewrite as

$$f_p(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^{m} C_i d_i^{\kappa} \qquad (7)$$

where

$d_i=0$, if constraint $g_i(\mathbf{x})$ is satisfied for i=1,…, m

$d_i=g_i(x)$, if constraint $g_i(\mathbf{x})$ is satisfied for i=1,…, m

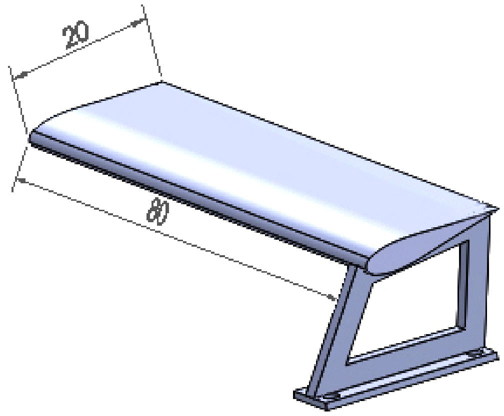
C_i is the penalty coefficients corresponding to i^{th} constraint and κ is here defined as 1.

4. Problem and Result

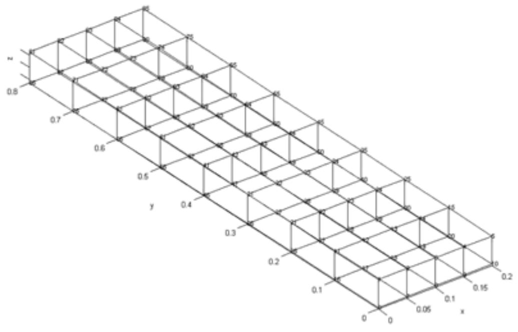
The aim of this work is to maximize the tip twist angle of the car-spoiler structure as shown in Fig. 2 whilst some structural and aeroelastic constraints are prevented. To reduce the time in calculation, only one flexible spoiler-wing of the airfoil spoiler as shown in Fig. 7 (a) can be analyzed to search for the maximum tip twist angle. The size of airfoil spoiler is 0.2(W) × 1.7(L) m. The spoiler wing consists of 5 ribs inside. The thickness of each rib is equal to 2 mm. The distance between both legs of spoiler is 0.1 m. The position of spar on the chord is set to 0.15 m or 75% of the chord. Like the flat spoiler, a part of spoiler between both legs was not considered in the optimization procedure. This design case was the sizing optimization problem with varying the thickness of spoiler skin and spar, the height at trapezoid spar at root and tip. Note that the trapezoid spar linearly varied from the root to tip. The AOA at the root and car speed were set to 8 degree and 160 km/hr, respectively. As shown in Fig. 7 (b), a FE model of spoiler wing-box consists of 90 nodes of the whole structure and 108 elements. The nodal points at root are fixed whilst the others are free.

The optimization problem consisted of one objective and constrained functions. The tip twist angle of a spoiler wing-box was optimized subject to deflection, stress, tip twist and critical speed (e.g. divergence and flutter speed) constraints. The structural and aeroelastic analysis were performed in the MATLAB program with developing FE code and PBIL algorithm. For PBIL approach, the parameters of population size, the number of bits for each design variable, the number of generations and the mut_shift were set as 10, 15, 400 and 0.2, respectively. The PBIL procedure stopped when the number of generations was reached. In addition, to transform the objective and constrained functions to the penalized objective function by using PBIL method, the penalty coefficient for the static penalty was set to five different levels of constraint violation. To adjust the magnitude of the penalty, the d_i can be treated to the form of normalized constraint with dividing by the upper bound in each constraint. Then, the penalty coefficient was defined as follows

$$C = \begin{cases} 0; \text{ if } d_i < 0 \\ 50000; \text{ if } 0 < d_i \leq 0.0005 \\ 30000; \text{ if } 0.0005 < d_i \leq 0.05 \\ 8000; \text{ if } 0.05 < d_i \leq 0.1 \\ 5000; \text{ if } d_i > 0.1 \end{cases} \quad (8)$$



(a) Model of spoiler wing



(b) FE model of spoiler wing-box

Fig. 7 Spoiler wing model

The constraints for the optimization were given as:

- the maximum of the AOA at the wing tip was set to 15 degree to ensure that the stall would not be occurred.
- the maximum of displacement at each node was set to 0.15 m.
- the maximum allowable stresses were the tensile and compression for aluminium 7075 as 505 MN/m^2 with the safety factor of 1.5.
- Both divergence and flutter were considered as a critical speed constraint. The critical speed constraint can be either divergence or flutter

speed depending upon the type of instability that happened first. The minimum of the allowed speed was set to 240 km/hr (or 66.7 m/s or 0.13 Mach number)

As the above discussion, four design variables consisted of the skin thickness of spoiler wing-box, the thickness of spar, the height of the trapezoid spar at the root and tip. Therefore, the optimization problem for the maximum tip twist angle can be stated in a mathematical form as follows

Max z(**x**) = tip twist angle of a spoiler wing-box (9)

Subject to

$$\frac{\theta}{15} - 1 \leq 0 \qquad (10)$$

$$\frac{\nu_j}{0.15} - 1 \leq 0;\ j = 1, 2, \ldots, 90 \qquad (11)$$

$$R_i - 1 \leq 0;\ j = 1, 2, \ldots, 90 \qquad (12)$$

$$1 - \frac{V_{cr}}{240} \leq 0 \qquad (13)$$

and bound constraints

$$0.0001 \text{ m} \leq x_1 \leq 0.005 \text{ m} \qquad (11)$$

$$0.00025 \text{ m} \leq x_2 \leq 0.005 \text{ m} \qquad (12)$$

$$0.015 \text{ m} \leq x_3 \leq 0.06 \text{ m} \qquad (13)$$

$$0.005 \text{ m} \leq x_4 \leq 0.06 \text{ m} \qquad (14)$$

where x_i was the i^{th} design variable, θ was the angle of attack at wing tip, ν was nodal displacements, R_i was the ratio of the nodal stress to the allowable stress at the i^{th} node and V_{cr} was a critical speed. Four design variables from x_1 to x_4 were the thickness of spoiler skin, the thickness of spar, the height of spar at the root and tip, respectively.

4. Optimization Results

This section shows the results of searching for the maximum tip twist angle of a spoiler wing-box subject to structural and aeroelastic constraints. PBIL procedure started with 10 arbitrary strings and recalculated until 100 generations reached. Search history of the best and average fitness values against the generation number are shown in Fig. 8. The results shows that the maximum tip twist angle is 4.98 degree with the constraints are accepted. Such results of the design case are given in Table 1. At the optimum, the 1^{st}, 3^{rd} and 4^{th} design variable attempt to reach the lower bounds; whilst, the 2^{nd} design variable of the thickness of spar reaches the upper bound.

Table 2 shows various values at the optimum design e.g. the position of elastic center, Tip twist angle at the wing tip, maximum displacement and stress, downforce on semi-wing-box, divergence and flutter speed. Fig. 9 shows that the maximum displacement of spoiler wing-box at the optimum design due to downforce at the car speed of 160 km/hr is 0.047 m. From Table 2, the difference of downforce between the flexural semi-wing and semi-rigid wing is 43.6 N or 87.2 N of the whole spoiler. Fig. 10 shows 4 natural frequencies and mode shapes of wing-box: 1^{st} bending at 38.1 Hz, 1^{st} torsion at 119.2 Hz, 2^{nd} bending at 199.1 Hz and 2^{nd} torsion at 361.8 Hz.

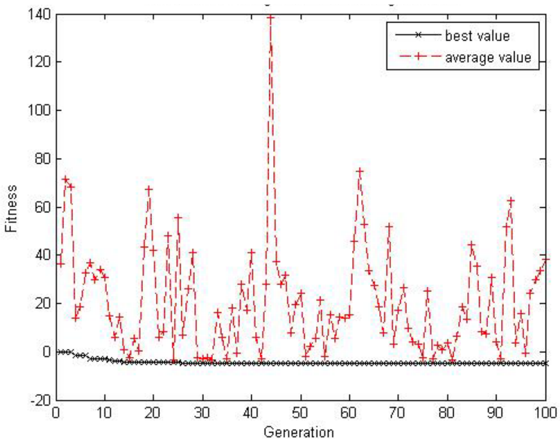


Fig. 8 Best and average fitness in each generation

Fig. 11 shows the comparative results at the optimum design of the airfoil spoiler with the flat spoilers at the AOA of 8 degree against the various car speeds from 0 to 200 km/hr. As shown in Fig. 11 (a) and (b), the changes of tip twist angle of the airfoil spoiler are higher than that of the flat-plate spoiler. Whilst the values of the maximum deflection of the flat plate spoiler is higher than the airfoil spoiler. Figure 11(c) shows the downforce on the flexible wing of the airfoil and flat plate spoiler. The result shows that the downforce on the airfoil spoiler is higher than that on the flat plate spoiler.

The results demonstrate that the airfoil spoiler provides the higher AOA and the larger downforce whilst its deflection is smaller when the results are compared with the flat plate spoiler. Therefore the airfoil spoiler with one fixed spar can get the benefit from structural deformation whilst the structural and aeroelastic failures are avoided.

Table 1 Optimum results

Design variables		Optimum
x_1	Skin thickness of spoiler wing-box (mm)	0.1
x_2	Spar thickness (mm)	1.3
x_3	Height of trapezoid spar at root (mm)	10.0
x_4	Height of trapezoid spar at tip (mm)	5.2
Max: Tip twist angle (degree)		4.98

Table 2 Results of various values at maximum tip twist angle of spoiler wing-box

The position of elastic center (% chord)	68.2
Tip twist angle at the wing tip (degree)	4.98
Maximum displacement (m)	0.047
Maximum stress (MN/m^2)	14.3
Downforce on a semi-wing (elastic structure) (N)	179.5
Downforce on semi-wing (rigid structure) (N)	135.9
Divergence speed (km/hr)	283.7
Flutter speed (km/hr)	240.0
Mass of semi-wing (kg)	0.16

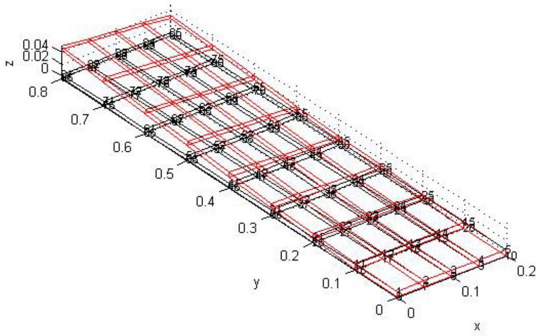
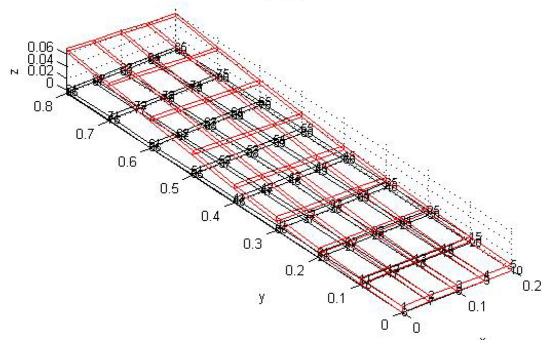
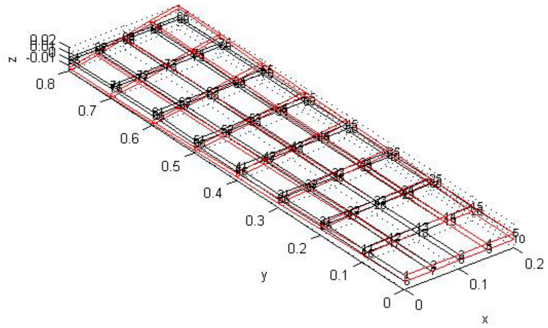


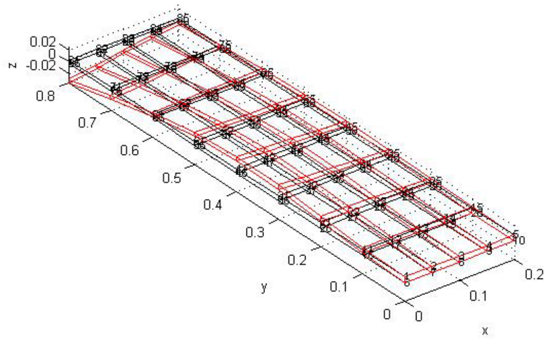
Fig. 9 Deflection of spoiler wing-box at car speed of 160 km/hr



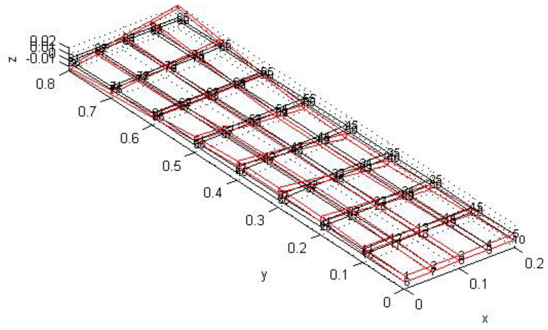
(a) 1st bending at 38.1 Hz



(b) 1st torsion at 119.2 Hz

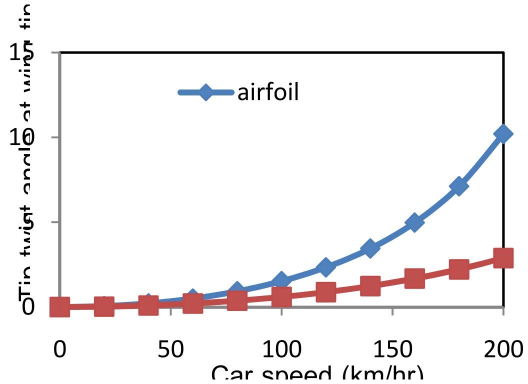


(c) 2nd bending at 199.1 Hz

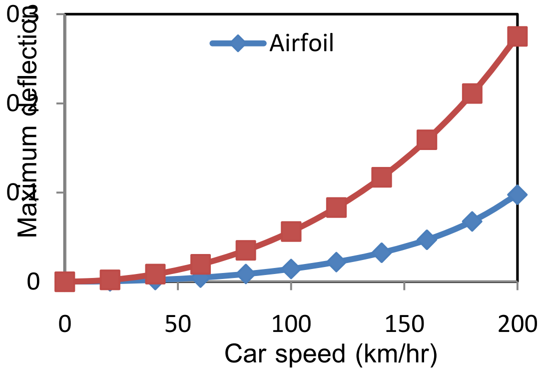


(d) 2nd torsion at 361.8 Hz

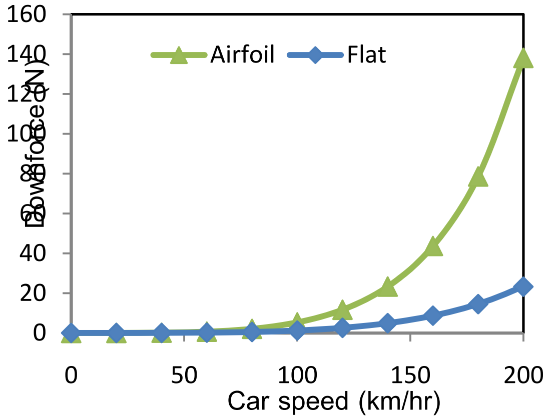
Fig. 10 Four mode shapes of spoiler wing-box



(a) Changes of tip twist angle



(b) Changes of maximum deflection



(c) Changes of different downforce between a flat and airfoil semi-wing

Fig. 11 Results of FEA at the root AOA of 8 degree with various car speeds for flat and airfoil spoiler

5. Conclusions

The rear spoiler is employed to control lift of car at the high speed with producing downforce. Typically, the structure of spoiler is very stiff to reduce the structural deformation. To get the benefit from the spoiler wing's deformation, the spoiler should get larger AOA at high speed and smaller AOA at low speed. The concept of aeroelastic spoiler was studied. One flat plate spoiler and one airfoil spoiler were compared. The flat plate spoiler increased the AOA and also offered the large deflection that was not required. To solve the large deflection and take advantage of wing twist, MDO technology was applied to the airfoil spoiler to maximize the tip twist angle. The results showed that the airfoil spoiler provided the higher AOA at high speed and lower AOA at low speed whilst its deflection was very small when they were compared with the results from the flat spoiler. Therefore, this paper shows the idea of aeroelastic spoiler how to use it in positive way from wing deformation and it will give the benefit to the automobile industry.

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