



An Approximate Analytical Modeling of Honeycomb Sandwich Structure Including the Effects of Imperfection, Core Thickness and Impact Damage

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Abstract

In this study an approximate buckling model for a sandwich structure consisting of a honeycomb core sandwiched between two layers is developed considering the geometric imperfection in the hexagonal structure core, core thickness and impact damage. These factors are important to be modeled for better overall design. Buckling stress with imperfection characteristics and impact damage are introduced by appropriate formulations. Core thickness affects the shear and normal stresses in honeycomb structures; the experimental data relating stresses to thickness is employed to form a relation. These terms is then integrated into the governing equation obtained from energy technique to improve the accuracy of the approximate analytical model. The results obtained are compared without these effects, and shows that the present model produces extremely accurate results.

Keywords: Modeling, honeycomb sandwich structure, imperfection, thickness, impact, energy principle

1. Introduction

Sandwich structures are laminated composite that are formed by bonding at least two different materials together. Sandwich structures can widely be used in virtually all fields including aircrafts, helicopters and nearly all military and space vehicles, solar panels, roofs, wind tunnels, light weight buses and many more. In many cases, these structures are employed to reduce stresses, are very stiff and have higher natural frequencies. Honeycomb sandwich structure is a sandwich structure in which a hexagonal core is sandwiched between two layers in the form of plates. The hexagonal core is usually manufactured from fiber glass, carbon reinforced plastic, aluminum or nomex and face

sheets are made from aluminum. Honeycomb structures are popular because of their low density and high strength to weight ratio.

Different instabilities can be found in a typical honeycomb structure that includes facing and transverse shear failure, local crushing of core, general buckling, shear crimping, face wrinkling and intra-cell buckling also referred as dimpling. These instabilities or failures are attributed to various loadings resulting in structural deformation that are harmful for stable structures. Failure or damage as a result of impact consists of multitude of factors such as configuration and thickness of face sheet, fabrication techniques, material geometry, interface properties between the face sheet and



the core, impact velocity and energy, indenter shape, temperature, boundary conditions and environmental factors. Hence it is essential for the scientists, engineers and materialist to design such a structure that might be able to cope all these factors caused by impact.

A variety of literature on analytical & experimental studies is available on buckling stresses of honeycomb sandwich structure. Almost every study shows that elastic as well as geometric properties of face sheet and honeycomb core along with dynamic/impact loading changes the critical buckling load. Differences between experimental and analytical studies have been found around 10-15%. Impact damage, initial stresses, change in properties of honeycomb structure due to imperfect manufacturing process are the main reasons for the variation between the two studies. Hoff and Mautner [1] developed the buckling model of a sandwich panel using energy method. This model gives a direct relationship between critical buckling stress and the Young's modulus of elasticity of face sheet, core and shear modulus of the core. The model is reproduced here as:

$$\sigma_c = c(E_f E_c G_c)^{\frac{1}{3}} \quad (1)$$

where E_f , E_c & G_c are the Young's modulus of face sheet, core & shear modulus of the core respectively, and c is the constant coefficient.

Plantema [2] modified the buckling model presented by Hoff and Mautner [1] by changing in the face sheet displacement. Hoff and Mautner model was then improved by improving the coefficient 'c' in the Eq. (1). He also suggested that the accuracy of the model can be increased up to 20% by taking into consideration the effects

of initial irregularities. These models are very effective but do not account for the core geometry and is dependent only on the properties of core and face sheet. Gough and de Bruyne [3], and Zenkert [4] used differential energy approach using Airy's Stress function and found an expression for the critical buckling load. Similarly Allen [5] used differential energy approach using sinusoidal function instead of Airy's function and related critical buckling stress with the Young's modulus of elasticity of face sheet, core, shear modulus of core and Poisson's ratio of the core. They assumed a Poisson's ratio of honeycomb core to be very small, and the variables of Eq. (1) of Hoff and Mautner model [1] were reduced with an improved coefficient relating the terms. All the presented models are incapable of showing the impact damage of core and face sheet, imperfections in the core geometry and face sheet. J. Chung and A.M Waas [6] explained that by using the classical beam bending theory the elastic strain energy stored in a unit cell can be found. He further explained that the in-plane elastic properties of core vary with shape and move isotropic to orthotropic form for a circular to elliptical core respectively. Their results showed that the in-plane properties such as E_x , G_{xy} , ν_{xy} increases and E_y decreases with the increase in eccentricity from 0 for a circle, and these properties also change with the change in shape of the cells. Lin and Huang [7] derived the expression for buckling model of honeycomb structure with varying thickness or plateau border. They obtained the critical buckling load and found that when the solid concentrated in the plateau region increases, the critical buckling stress



decreases. Mei-Yi Yang et al. [8] employed energy techniques to find the critical buckling load of a honeycomb structure with dual imperfection of curved edges over varying thickness of the core in plateau region. They showed that with increasing plateau border and increasing curvature of cell wall the critical buckling stress decreases. The main advantage of this technique is that they included the effect of dual deformation on buckling stress of a core only, which considerably affects the overall performance of the model. J.Gustin et al. [9] tested the core sandwich structures experimentally and studied the effects of impact like deformation or stresses induced. They presented force versus time response curves for core with varying impact energy, and compared their results with a simplified mass spring system, and showed that the experimental results were 50% of what was predicted by the mass spring system. It means the model needs to be further improved for analyzing impact behavior. Donald and Herbert [10] studied impact of different caliber bullets on an aircraft panel modeled as a honeycomb material with change in orientation of the panel. He presented results showing the loss in flexural and peel strength in the region of bullet hole with varying bullet caliber and changing bullet impact angle. Yi-Ming Jen and Li-Yen Chang [11] performed an experimental study on three different types of honeycomb sandwich structures with varying face sheet thickness. They estimated the effects of thickness of face sheet on the bending fatigue strength, and presented results graphically relating interfacial peeling and shear stresses with the thickness of face sheet. A

decrease in interfacial peeling and shear stresses with the increase in face sheet thickness was observed.

Above all, there are no such single analytical model exists in the past literature that covers the dual deformation of hexagonal core, impact damage and core & face sheet thickness over the critical buckling load. In this study, a new form of the analytical model is proposed by taking into consideration of all these three effects for better results. The analytical modeling is discussed in the subsequent section followed by results and some useful conclusions.

2. Analytical Modeling of Honeycomb Sandwich Structures

Figure 1 shows a honeycomb sandwich structure bonded together between two layers or face sheets. Both honeycomb structure and face sheets are made up of aluminium. In the literature, the researchers gave the relationship for the shear buckling of a sandwich panel without considering the geometric imperfections, impact damage and effect of core and face sheet thickness. The inclusion of these terms provides an extremely accurate representation of the model and more refine solution of the system.

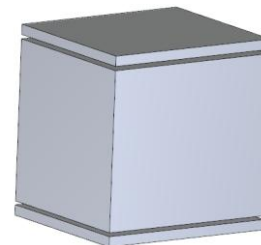


Fig. 1 Two plate honeycomb sandwich structure

To start with, Hoff and Mautner [1] model as described earlier is reproduced here

$$\sigma_c = c(E_f E_c G_c)^{\frac{1}{3}} \quad (2)$$



This model however didn't include the aforesaid effects and is not demonstrated accurately a shear buckling of a sandwich panel. Numerous studies was carried out for further refinement of this model and proposed a more advanced and refined forms. In all these cases the only modification in coefficient 'c' can be seen and is tabulated in Table. 1 for different authors.

Table. 1 Coefficients of Analytical Model

Analytical Model	Coefficient of Analytical Model [Eq. 2]	
	Initial	Revised
Hoff and Mautner [1]	0.91	0.8255
Plantema [2]	0.9	
Gough and De Bryne [3]	0.79	
Zenkert [4]	0.78	
Allen [5]	Upper Bound	Lower Bound
	0.78	0.63

The decreasing trend in the value of c can be observed.

2.1 Effect of Geometrical Imperfection

Analytical and experimental results of different studies shows certain variations that is attributed to manufacturing defects and imperfections in the structure which causes 10-20% drop in failure loads. Secondly, these small initial imperfections trigger a core compression or core to face sheet failure. Plantema [2] suggested that initial irregularities are likely to reduce the wrinkling stress by up to 20%. If the structure is free of imperfection, it might be failed at a marginally higher load than the eigenvalue failure load. In reality no structure is free of imperfections and irregularities. Hence, the effect of imperfection should be accounted for and must be designed in the analytical model.

Mei-Yi Yang et al. [8] determined the relation showing the geometric imperfection, and

developed the equation that can be written here as:

$$\sigma = \frac{E_s \alpha}{\sqrt{3}l^2} \tag{3}$$

The term α is represented by

$$\alpha = \left(\frac{3}{2} \left(\int_0^{\frac{R_p}{2}} \frac{2}{t(x)^3} dx + \int_0^\phi \frac{2R}{t_0^3} d\xi \right) + \frac{12}{L^2} \left(\int_0^{\frac{R_p}{2}} \frac{2}{t(x)^3} \left(\frac{L}{2} - x\right)^2 dx + \int_0^\phi \frac{2R^3}{t_0^3} [\sin(\phi - \xi)]^2 d\xi \right) - \frac{12}{L^2} \left(\int_0^\phi \frac{2R^3}{t_0^3} [\cos(\phi - \xi) - \cos\phi]^2 d\xi \right) \right)^{-1} \tag{4}$$

where E_s is the Young's modulus of solid cell edges, l is the cell length, R_p is the radius of curvature of plateau borders, $t(x)$ is the cell thickness, R is the curvature of outside Plateau borders, ϕ is the angle between the two lines drawn from the center of curvature to both ends of the constant thickness region of each circular cell edge, x is measured along the neutral axis of each cell, ξ is the small angle and L is the effective length. All these parameters can easily be illustrated in Fig. 2.

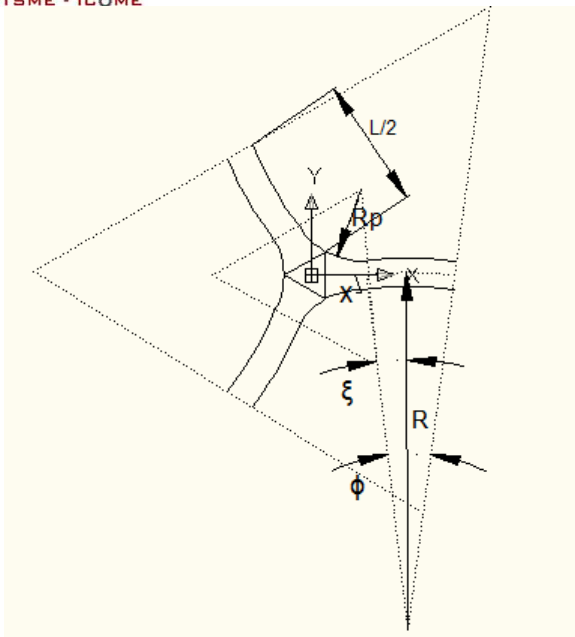


Fig. 2 Unit edge showing dual imperfections [8]
Density ratio (ρ^*) and volume fraction (v_f) are usually used to quantify the geometric imperfection in the core. Therefore, taking relations from ref [8] for analyzing the behavior of the structure the density ratio and volume fraction are given by

$$\rho^* = 2/3 \left[\left(2 - \frac{\pi}{\sqrt{3}} \right) \left(\frac{R_p}{l} \right)^2 + 2 \left(\frac{R_p}{l} \right) \left(\frac{t_0}{l} \right) + \frac{1}{2} \left(\frac{t_0}{l} \right)^2 - 2\phi \csc \phi \cos \phi \left(\frac{R_p}{l} \right) \left(\frac{t_0}{l} \right) - \phi \csc \phi \cos \phi \left(\frac{t_0}{l} \right)^2 + \sqrt{3}\phi \csc \phi \left(\frac{t_0}{l} \right) \right] \quad (5)$$

and,

$$v_f = \frac{2}{3\rho^*} \left[\left(2 - \frac{\pi}{\sqrt{3}} \right) \left(\frac{R_p}{l} \right)^2 + 2 \left(\frac{R_p}{l} \right) \left(\frac{t_0}{l} \right) + \frac{1}{2} \left(\frac{t_0}{l} \right)^2 \right] \quad (6)$$

By setting $t(x) = t_0$, $L = l$, $R_p = 0$ and $\phi = 0$ in Eq. (3), we get stress without imperfection i.e.

$$\sigma_o = \frac{2E_s t^3}{5\sqrt{3}l^3} \quad (7)$$

Dividing Eq. (3) with Eq. (7), we get the ratio of stress with and without imperfection

$$\beta = \frac{\sigma}{\sigma_o} = \frac{5l}{2t^3} \alpha \quad (8)$$

To examine the effect of core and face sheet separately let take a very useful relation from ref. [12]

$$P_{cri} = \frac{\pi^2 E_f t_f^3}{12L_1^2} + \frac{L_1 \sqrt{G_c E_c}}{\pi} \quad (9)$$

where P_{cri} is critical buckling load per unit thickness, E_f is Young's modulus of face sheet, t_f is the thickness of face sheet, L_1 is the half critical wavelength, G_c is the transverse shear modulus of the core and E_c is Young's modulus of core.

An expression for the critical wrinkling half wavelength can be obtained by differentiating P_{cri} with respect to L_1 , [12] we get,

$$L_1 = 1.73 t_f \left(\frac{E_f^2}{E_c G_c} \right)^{1/6} \quad (10)$$

Substituting the value of critical wrinkling half wavelength from Eq. (10) into the Eq. (9), we get,

$$\sigma_{cri} = 0.275 (E_f E_c G_c)^{1/3} + 0.55 (E_f E_c G_c)^{1/3} \quad (11)$$

Eq. (11) consists of two parts. The first part represents the face sheet whereas second part of this equation shows the core without imperfection.

Substituting Eq. (8) into the second part of the Eq. (9), we get,

$$\sigma_{cri} = 0.275 (E_f E_c G_c)^{1/3} + 0.55 \beta (E_f E_c G_c)^{1/3} \quad (12)$$

Eq. (10) shows the effects of geometric imperfection on critical stress.

2.2 Effect of Impact Damage, and Thickness of Core & Face Sheet

Two types of damage occur in a typically sandwich structure. In the first type impacts puncture the skin or face sheet and must be repaired straight away, while in the second type impacts crush the core directly below the face sheet and cause a large, barely visible impact dent (BVID) without the failure of face sheet. Impact damages have been shown to greatly affect the load carrying capacity of the component,



causing panels to fail at lower loads than expected. Aitken [14] showed that impact damage larger than 60mm reduces the panels load carrying capacity by 50%. He further explained that instead of these reductions, the panels may still be able to meet the design requirements as most components are designed for stiffness and not for strength. For these reasons, it is essential to determine the effect of impact as it decreases the load carrying capacity of sandwich panel.

The effects of impact or impulses on the critical stress for the buckling of a honeycomb sandwich structure were analyzed by Wang [14] using free drop and shock absorbing principle. He obtained a relationship of energy absorption per unit volume with relative density and thickness of the honeycomb structure. The energy absorbed was found to increase with increase in relative density of paper honeycomb. The results showed inconsistent behavior for increasing thickness of the honeycomb structure. His proposed relation can be written as:

$$E = \frac{\sigma_m H}{GT} \quad (13)$$

where E is the impact energy per unit volume, σ_m is the maximum shear stress, G is the ratio of peak acceleration to gravitational acceleration, T is the thickness of cushioning material and H is the drop height.

The thickness of sandwich panel or plate has a great impact on its acceleration transmissibility. With the increase of thickness, the acceleration transmissibility decreases and cushioning effect of sandwich panel increases. A thicker face sheet or sandwich panel is more resistant to impact damage and absorbs most of the energy due to increase in bending stiffness. The damage is

either seen in the face sheet or at the core/face sheet interface. In case of thin face sheet most of the energy is absorbed by the core and the damage is seen in the form of multiple fractures and crease lines through the damage depth.

Wang [14] further illustrated experimentally the relationship for maximum stress and thickness of face sheet. The generalized equation of maximum shear stress can be obtained in the form of constants A , a , B , b , D and d which depend on the material used. The curve fitting technique is used, which leads the following equation for maximum shear stress:

$$\sigma_m = Ae^{-at} + Be^{-bt} - De^{-dt} \quad (14)$$

where t is the thickness of face sheet. Wang [10] used aluminum as the specimen material the coefficient obtained are $A=2.021$, $a=1.289$, $B=7.577$, $b=1.953$, $D=7.763$ and $d=2.433$

Substituting the value of σ_m from Eq. (14) into Eq. (13), Eq. (13) can be then written in term of critical stress since E is the impact energy per unit volume, therefore

$$\sigma_{cri} = \frac{(Ae^{-at} + Be^{-bt} - De^{-dt})H}{GT} \quad (15)$$

Eq. (15) represents a relationship between critical stress, impact damage and core thickness.

A complete form can then be obtained with the inclusion of these three effects lead the following form:

$$\sigma_{cri} = \frac{0.275(E_f E_c G_c)^{\frac{1}{3}} + 0.55\beta(E_f E_c G_c)^{\frac{1}{3}}}{(Ae^{-at} + Be^{-bt} - De^{-dt})H} \cdot GT \quad (16)$$

3. Results and Discussions

As seen, the critical stress can be calculated for a modified form represented in Eq. (16). Figures 3-5 represent the analytical curves



obtained from the formulation explained in the previous sections.

Figure 3 shows that with the increase in angle, ϕ and the volume fraction the critical stresses decrease and the structure would buckle earlier due to decrease in thickness and increase in R_p and R which in turn increases geometric imperfection. There will be a less imperfection with the increase in density ratio as compare to more imperfection in case of increase in volume fraction which can be seen in Figure 3. This figure also illustrates the effect of change in face sheet thickness on critical stress, and observed that both curves for given parameters ($v_f = 0.02$, $\rho^* = 0.015$, and $t=1.5\text{mm}$ & $t=1.0\text{mm}$ for the other) coincide with each other. This shows a very negligible effect as compared to other parameters.

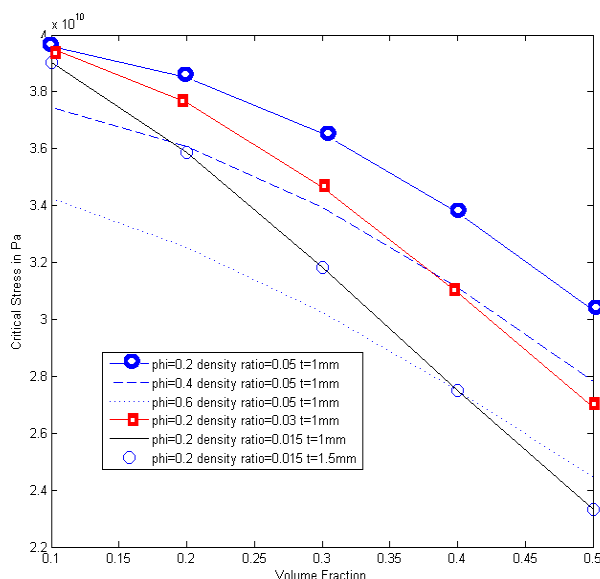


Fig. 3 Variations in critical stress with change in parameters (ϕ , v_f , ρ^* and t)

Figure 4 shows that the results are much improved with the addition of geometric imperfection into the proposed analytical model.

The percentage change in improvement is caused by the factor α which defines the dual imperfection. For an ideal core α is one and approaches to zero with the increase in the imperfection of core. The %change in results vary from 70% for α approaching zero and 0% for $\alpha=1$ that extreme is practically impossible to give the core hexagonal shape with straight cell edges and no variation in core thickness.

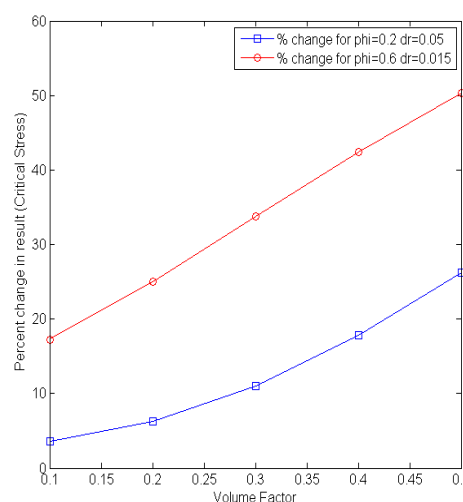


Fig. 4 Percent Change in Results between existing Eq. (11) and proposed model Eq. (12)

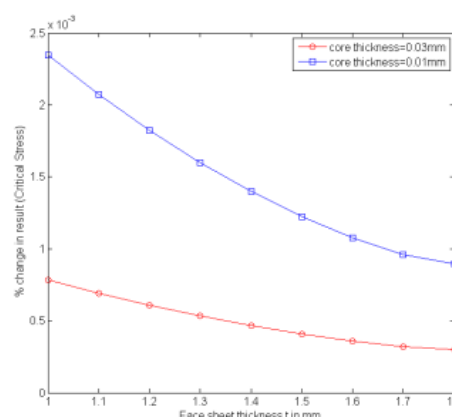


Fig. 5 Change in Results from model presented in Eq. (11) due to impact energy and face sheet

Figure 5 depicts the variations when considering the face sheet thickness and impact energy. A very small %change almost in the range of



(0.00025-0.0025)% is observed. The percentage changes decreases with the increase in both cushioning thickness and face sheet thickness as expected.

4. Conclusions

A new form of the analytical model is obtained for a sandwich structure consists of honeycomb core sandwiched between two plates. The effects of dual imperfection in core, face sheet thickness and impact energy are added into the model in order to get a more tactical solution. The model indicates a variation from 0 to 70% in the critical stresses with dual imperfection and similarly (0.00025 to 0.0025)% changes with the addition of impact energy and face sheet thickness.

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